Chapter 8

Introducing Inferential Statistics

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CHAPTER OBJECTIVES – STUDENTS SHOULD BE ABLE TO:

- Explain the difference between descriptive and inferential statistics.
- Define the central limit theorem and explain why it is important to the world of inferential statistics.
- List the steps in completing a test of statistical significance.
- Discuss the basic types of statistical tests and how they are used.
- Explain Type I and Type II errors in null hypothesis testing.
- Discuss the distinction between statistical significance and meaningful significance.
CHAPTER OVERVIEW

- Say Hello to Inferential Statistics
- The Idea of Statistical Significance
- Tests of Significance
- Significance Versus Meaningfulness
- Meta-analysis
SAY HELLO TO INFERENTIAL STATISTICS

- Descriptive statistics provide basic measures of a distribution of scores
- Inferential statistics allow inferences to a larger population from the sample
HOW INFERENCE WORKS

1. Representative samples from two groups are selected
2. Participants are tested
3. Means from each group are compared
4. Researchers conclude that measured differences between groups either
   a. Result from chance, or
   b. Reflect true differences
5. A conclusion is drawn regarding the role group membership plays in observed differences
THE ROLE OF CHANCE

- Chance is the first explanation for observed differences
  - Chance is unexplained variability

- The goal of science is to
  - Control sources of variability, thus
  - Reducing the role of chance as an explanation
THE CENTRAL LIMIT THEOREM

- The means of samples drawn from a population will be normally distributed.
- This is so regardless of the shape of the population distribution.
- This demonstrates the power of inference.
AN EXAMPLE OF THE CENTRAL LIMIT THEOREM

![Graphs](image_url)
THE IDEA OF STATISTICAL SIGNIFICANCE

- Because sampling is imperfect
  - Samples may not ideally match the population, and
- Because hypotheses cannot be directly tested
  - Inference is subject to error
The degree of risk that you are willing to take that you will reject a null hypothesis when it is actually true
## MAKING A DECISION

<table>
<thead>
<tr>
<th>If You…</th>
<th>When the Null Hypothesis Is Actually…</th>
<th>Then You Have…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject the null hypothesis</td>
<td>True (there really are no differences)</td>
<td>Made a Type I Error</td>
</tr>
<tr>
<td>Reject the null hypothesis</td>
<td>False (there really are differences)</td>
<td>Made a Correct Decision</td>
</tr>
<tr>
<td>Accept the null hypothesis</td>
<td>False (there really are differences)</td>
<td>Made a Type II Error</td>
</tr>
<tr>
<td>Accept the null hypothesis</td>
<td>True (there really are no differences)</td>
<td>Made a Correct Decision</td>
</tr>
</tbody>
</table>
TYPE I AND TYPE II ERRORS

• The probability of making a Type I error
  – Set by researcher
    • e.g., .01 = 1% chance of rejecting null when it is true
    • e.g., .05 = 5% chance of rejecting null when it is true
  – Not the probability of making one or more Type I errors on multiple tests of null!

• The probability of making a Type II error
  – Not directly controlled by researcher
  – Reduced by increasing sample size
HOW A TEST OF SIGNIFICANCE WORKS

- Each type of null hypothesis is tested with a particular statistic
- Each statistic is characterized by a unique distribution of values that are used to evaluate the sample data
1. State the null hypothesis

   \[ H_0: \mu_1 = \mu_2 \]

2. Establish significance level

   e.g., \( p = .05 \)
   e.g., \( p = .01 \)
3. Select appropriate test statistic
4. Compute test statistic (obtained value)
5. Determine value needed to reject null (critical value), which depends on
   a. Level of significance chosen (e.g., \( p = 0.5 \))
   b. Degrees of freedom (based on sample size)
6. Compare obtained value to critical value
7. If obtained value > critical value, reject null
8. If obtained value \( \leq \) critical value, accept null
t–TEST FOR INDEPENDENT MEANS

Used to test null hypothesis when two independent, unrelated groups are compared  
E.g., Chen and Stevenson (1989)

1. State null
2. Establish level of risk
3. Select test statistic
4. Compute value
5. Determine critical value
6. Compare obtained value

1. $H_0: \mu_{1980} = \mu_{1984}$
2. $p = .05$
3. t–test
4. 2.00
5. 1.980
6. $2.00 > 1.980; p < .05$

<table>
<thead>
<tr>
<th>Degrees of Freedom</th>
<th>.05 Level of Significance</th>
<th>.01 Level of Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>2.021</td>
<td>2.704</td>
</tr>
<tr>
<td>60</td>
<td>2.00</td>
<td>2.660</td>
</tr>
<tr>
<td>120</td>
<td>1.980</td>
<td>2.617</td>
</tr>
</tbody>
</table>

WHAT DOES $t_{120} = 2.00, p < .05$ Really Mean?

- $t =$ type of test
- $120 =$ degrees of freedom
  - (related to sample size)
- $2.00 =$ obtained value of t-test
- $p =$ probability
- $.05 =$ level of significance
  - (Type I error rate)
A NEW TWIST TO \( p < .05 \)

- A statement of probability, e.g., \( p < .05 \) indicates that the probability of making a Type I error on a test is less than .05

- But SPSS and other data analysis software compute exact probabilities, e.g., \( p = .0375 \)
# LOOKING AT DIFFERENCES BETWEEN GROUPS

<table>
<thead>
<tr>
<th>Differences Between Groups</th>
<th>The Null Hypothesis</th>
<th>The Statistical Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is there a difference between the means of two <em>unrelated</em> groups?</td>
<td>$H_0: \mu_{\text{group}<em>1} = \mu</em>{\text{group}_2}$</td>
<td>t-test for independent means</td>
</tr>
<tr>
<td>Is there a difference between the means of two <em>related</em> groups?</td>
<td>$H_0: \mu_{\text{group}<em>{1a}} = \mu</em>{\text{group}_{1b}}$</td>
<td>t-test for dependent means</td>
</tr>
<tr>
<td>Is there an <em>overall</em> difference between the means of <em>three</em> groups?</td>
<td>$H_0: \mu_{\text{group}<em>1} = \mu</em>{\text{group}<em>2} = \mu</em>{\text{group}_3}$</td>
<td>Analysis of variance</td>
</tr>
</tbody>
</table>

t-TEST FOR DEPENDENT MEANS

1. State null
2. Establish level of risk
3. Select test statistic
4. Compute value
5. Determine critical value
6. Compare obtained value

1. \( H_0: \mu_{1A} = \mu_{1B} \)
2. \( p = .01 \)
3. t-test
4. 2.581
5. 2.771
6. 2.581 < 2.771; \( p > .01 \)

<table>
<thead>
<tr>
<th>Degrees of Freedom</th>
<th>Level of Significance for a One-Tailed Test</th>
<th>Level of Significance for a Two-Tailed Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.05</td>
<td>.025</td>
</tr>
<tr>
<td></td>
<td>.10</td>
<td>.05</td>
</tr>
<tr>
<td>26</td>
<td>1.706</td>
<td>2.056</td>
</tr>
<tr>
<td>27</td>
<td>1.703</td>
<td>2.052</td>
</tr>
<tr>
<td>28</td>
<td>1.701</td>
<td>2.048</td>
</tr>
<tr>
<td>29</td>
<td>1.699</td>
<td>2.045</td>
</tr>
<tr>
<td>30</td>
<td>1.697</td>
<td>2.042</td>
</tr>
</tbody>
</table>
# LOOKING AT RELATIONSHIPS BETWEEN VARIABLES

<table>
<thead>
<tr>
<th>Relationships Between Variables</th>
<th>The Null Hypothesis</th>
<th>The Statistical Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is there a relationship between two variables?</td>
<td>$H_0: \rho_{xy} = 0$</td>
<td>t-test for significance of the correlation coefficient</td>
</tr>
<tr>
<td>Is there a difference between two correlation coefficients?</td>
<td>$H_0: \rho_{ab} = \rho_{cd}$</td>
<td>t-test for the significance of the difference between correlation coefficients</td>
</tr>
</tbody>
</table>
MULTIVARIATE ANALYSIS OF VARIANCE (MANOVA)

- Simultaneously tests differences between groups on multiple dependent variables, but
  - Because dependent variables might be related
  - True Type I Error rate is inflated

\[
1 - (1 - \alpha)^k
\]

\[\alpha = \text{Type I error rate}\]

\[k = \text{number of pairwise comparisons}\]

- So, MANOVA takes these possible relationships into account
A factor groups several related measures into one construct

The new construct is treated as a dependent variable

This technique allows a researcher to more efficiently examine how these sets of variables are related
SIGNIFICANCE VERSUS MEANINGFULNESS

- Statistical significance refers to the
  - Probability that chance influenced observed differences
  - It does NOT refer to the meaningfulness or “importance” of observed differences

- Statistical significance must be interpreted within a larger context
META-ANALYSIS

- Compares the results of multiple independent studies that have examined the same conceptual, dependent variable.
- Allows examination of trends and patterns that may exist in many different groups in many different studies.
HOW META-ANALYSES ARE DONE

1. An adequate sample of studies is collected
2. Results from these studies are converted to a common measure—usually effect size
3. Important aspects of the study are coded
4. Descriptive and correlational techniques are used to look for trends or common patterns in the outcomes of the group of studies
HAVE WE MET OUR OBJECTIVES?
CAN YOU:

- Explain the difference between descriptive and inferential statistics?
- Define the central limit theorem and explain why it is important to the world of inferential statistics?
- List the steps in completing a test of statistical significance?
- Discuss the basic types of statistical tests and how they are used?
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